# Section 3.9: Derivatives of Exponential and Logarithmic Functions

## Derivative of the Exponential Function

**Derivative of the Natural Exponential Function**

Let be the natural exponential function. Then

.

In general,

.

Media: Watch these [video](https://youtu.be/1Zbelm56VKA)1 and [video2](https://youtu.be/B8olpqejnjE) examples on the derivative of an exponential function.

Examples

1. Find the derivative of each of the following:
2. A colony of mosquitoes has an initial population of . After days, the population is given by . Show that the ratio of the rate of change of the population, , to the population, is constant.

## Derivative of the Logarithmic Function

Now that we have the derivative of the natural exponential function, we can use implicit differentiation to find the derivative of its inverse, the natural logarithmic function.

**The Derivative of the Natural Logarithmic Function**

If and , then

More generally, let be a differentiable function. For all values of for which , the derivative of is given by

Media: Watch these [video](https://youtu.be/BYFiDW1_pxk)1 and [video2](https://youtu.be/R9c-vEuYjj4) examples on the derivative of logarithmic functions.

Examples: Find the derivative of each of the following:

Now that we can differentiate the natural logarithmic function, we can use this result to find the derivatives of and for .

**Derivatives of General Exponential and Logarithmic Functions**

Let , and let be a differentiable function.

1. If, , then

.

1. If , then

.

Media: Watch these [video](https://youtu.be/P3ek1zjlG30)1 and [video2](https://youtu.be/HORYwxL0c34) examples on derivatives of exponentials and logs with other bases.

Examples

1. Find the slope of the line tangent to the graph of at .
2. Find the derivative of .

## Logarithmic Differentiation

Logarithmic differentiation allows us to differentiate any function of the form . It can also be used to convert a very complex differentiation problem into a simpler one.

**How to Solve Using Logarithmic Differentiation**

1. To differentiate using logarithmic differentiation, take the natural logarithm of both sides of the equation to obtain .
2. Use properties of logarithms to expand as much as possible.
3. Differentiate both sides of the equation. On the left we will have .
4. Multiply both sides of the equation by to solve for .
5. Replace by .

Media: Watch these [video](https://youtu.be/ivMj1gl3UsE)1 and [video2](https://youtu.be/SqB-MaegTkY) examples on logarithmic differentiation.

Examples: Find the derivative of each of the following.